1505: Update on lattice QCD with domain wall quarks

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Abstract. Using domain wall fermions, we estimate $B_K(\mu \approx 2\,\mathrm{GeV}) = 0.602(38)$ in quenched QCD which is consistent with previous calculations. We also find ratios of decay constants that are consistent with experiment, within our statistical errors. Our initial results indicate good scaling behavior and support expectations that $O(a)$ errors are exponentially suppressed in low energy ($E \ll a^{-1}$) observables. It is also shown that the axial current numerically satisfies the lattice analog of the usual continuum axial Ward identity and that the matrix element of the four quark operator needed for B_K exhibits excellent chiral behavior.

1 Introduction

We recently reported[1, 2] on calculations using a new discretization for simulations of QCD, domain wall fermions (DWF) [3, 4], which preserve chiral symmetry on the lattice in the limit of an infinite extra 5th dimension. There it was demonstrated that DWF exhibit remarkable chiral behavior[1] even at relatively large lattice spacing and modest extent of the fifth dimension.

In addition to retaining chiral symmetry, DWF are also "improved" in another important way. In the limit that the number of sites in the extra dimension, N_s, goes to infinity, the leading discretization error in the effective four dimensional action for the light degrees of freedom goes like $O(a^2)$. This theoretical dependence is deduced from the fact that the only operators available to cancel $O(a)$ errors in the effective action are not chirally symmetric [5, 2]. For finite N_s, $O(a)$ corrections are expected to be exponentially suppressed with the size of the extra fifth dimension. Our calculations for B_K show a weak dependence on a that is easily fit to an a^2 ansatz. Preliminary results for the ratios f_π/m_ρ and f_k/f_π indicate good scaling behavior as well.

We use the boundary fermion variant of DWF developed by Shamir. For details, consult Kaplan[3] and Shamir[4]. See Ref. [6] for a discussion of the $4d$ chiral Ward identities (CWI) satisfied by DWF. Our simulation parameters are summarized in Table 1.

2 Results

We begin with the numerical investigation of the lattice PCAC relation. The CWI are satisfied exactly on any configuration since they are derived from the

Table 1. Summary of simulation parameters. M is the five dimensional Dirac fermion mass, and m is the coupling between layers $s = 0$ and $N_s - 1$.

$6/g^2$	size	M	$m(\#\ \text{conf})$
5.85	$16^3 \times 32 \times 14$	1.7	0.075(34) 0.05(24)
6.0	$16^3 \times 32 \times 10$	1.7	0.075(36) 0.05(39) 0.025(34)
6.3	$24^3 \times 60 \times 10$	1.5	0.075(11) 0.05(15) 0.025 (22)

corresponding operator identity. We checked this explicitly in our simulations. In the asymptotic large time limit, we find for the usual PCAC relation

$$2 \sinh \left(\frac{am_\pi}{2} \right) \frac{\langle A_\mu | \pi \rangle}{\langle J_5 | \pi \rangle} = 2m + 2 \frac{\langle J_{5q} | \pi \rangle}{\langle J_5 | \pi \rangle}, \tag{1}$$

which goes over to the continuum relation for $am_\pi \ll 1$ and $N_s \to \infty$ (see Ref. [6] for operator definitions). The second term on the r.h.s. is anomalous and vanishes as $N_s \to \infty$. It is a measure of explicit chiral symmetry breaking induced by the finite 5th dimension. At $6/g^2 = 6.0$ and $N_s = 10$ we find the l.h.s. of Eq.1 to be 0.1578(2) and 0.1083(3) for $m = 0.075$ and 0.05, respectively. The anomalous contributions for these two masses are $2 \times (0.00385(5)$ and $0.00408(12))$, which appear to be roughly constant with m. Increasing N_s to 14 at $m = 0.05$, the anomalous contribution falls to $(2\times)\ 0.00152(8)$ while the l.h.s. is 0.1026(6), which shows that increasing N_s really does take us towards the chiral limit.

Next we investigate the matrix element of the four quark operator O_{LL} which defines B_K. $\langle K | O_{LL} | \bar{K} \rangle$ vanishes linearly with m in the chiral limit in excellent agreement with chiral perturbation theory (Fig. 1).

In Fig. 2 we show the kaon B parameter at each value of $6/g^2$ versus af_π which is used to set the lattice spacing. The results for B_K depend weakly on $6/g^2$, and are well fit to a pure quadratic in a. We find $B_K(\mu = a^{-1}) = 0.602(38)$ in the continuum limit. This value is already consistent with previous results[7] though it does not include the perturbative running of B_K to a common energy scale. This requires a perturbative calculation to determine the scale dependence of O_{LL}, which has not yet been done[8]. From Table 2, the energy scale at $6/g^2 = 6.0$ is roughly 2 GeV.

At $6/g^2 = 6.0$, we have also calculated B_K using the partially conserved axial current $A_\mu^a(x)$ (and the analogous vector current). This point split conserved current requires explicit factors of the gauge links to be gauge invariant. Alternatively a gauge non-invariant operator may be defined by omitting the links; the two definitions become equivalent in the continuum limit. Results for the gauge non-invariant operators agree within small statistical errors with those obtained with naive currents, Fig. 2(see Ref.[6, 1] for operator definitions). The results for the gauge invariant operators are somewhat
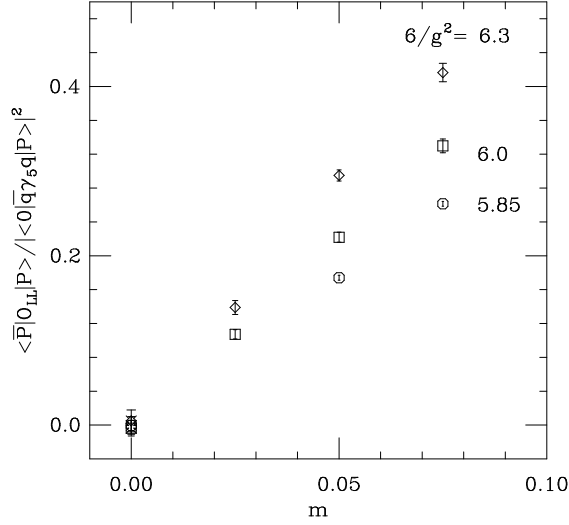
Fig. 1. The matrix element $\left\langle \bar{P}|O_{LL}|P \right\rangle$ *vs. m*. *m* is proportional to the quark mass in lattice units. $N_s = 10$ ($6/g^2 = 6.0$, 6.3) and 14 ($6/g^2 = 5.85$).

larger: $B_K^{inv}(\mu = a^{-1}) = 0.857(20)$ and $0.946(28)$ at $m = 0.05$ and 0.075, respectively. A similar situation holds in the Kogut-Susskind case where it was shown that the gauge invariant operators receive appreciable perturbative corrections which bring the two results into agreement [9].

Table 2. Lattice spacing and decay constant summary.

$6/g^2$	$a^{-1}(m_\rho)$	$a^{-1}(f_\pi)$	f_π/m_ρ	f_K/f_π	N_s
5.85	1.49(29)	1.62(27)	0.154(40)	1.206(15)	14
6.0	1.89(14)	2.06(15)	0.155(11)	1.205(15)	10
6.3	2.96(25)	3.24(27)	0.155(19)	1.14(14)	10

Using Eq. 1, neglecting the anomalous contribution, and using the definition of the decay constant, we can determine the pseudoscalar decay constant from the measurement of $\langle 0|J_5^a|P \rangle$. The results are summarized in Table 2. As with B_K, the estimates of the physical ratios f_π/m_ρ and f_K/f_π indicate good scaling behavior. They are also consistent with experiment, within rather large statistical errors. The errors in Table 2 are crude estimates derived using the statistical uncertainties only; the small sample size precludes
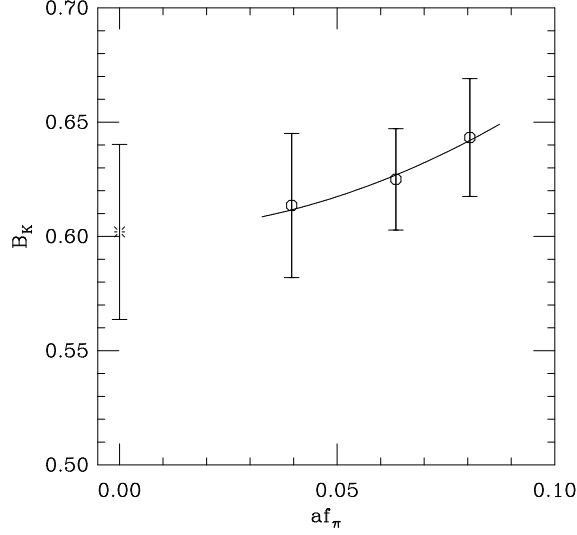
Fig. 2. The kaon B parameter.

us from properly accounting for the correlations in the data. All of the results are so called effective values, calculated from the two-point correlators without χ^2 fits and averaged over suitable plateaus. In the last column at the two larger couplings, an attempt has been made to carry out a more sophisticated jackknife error analysis all the way through to the final ratio.

While all of the above results indicate good scaling, they must be checked further with improved statistics and a fully covariant fitting procedure. Also systematic effects like finite volume still need to be investigated. Only then can the continuum limit can be reliably taken. We note that a recent precise calculation using quenched Wilson quarks by the CP-PACS collaboration yields values of f_π and f_K in the continuum limit that are inconsistent with experiment [10].

In Fig. 3 we show the pion mass squared as a function of m. For $N_s = 10$ the data at $6/g^2 = 6.0$ and 6.3 are consistent with chiral perturbation theory. However, at $6/g^2 = 5.85$, m_π^2 extrapolates to a positive non-zero value in the chiral limit for $N_s = 10$ and 14. There is a large downward shift in the line as N_s goes from 10 to 14, but it still does not pass through the origin. However, at $N_s = 18$, m_π^2 extrapolates to -0.004(19) at $m = 0$. The anomalous contribution on the r.h.s. of Eq. 1 drops from 0.0098(5) to 0.0038(2) as N_s varies from 10 to 18. It is interesting to note that at $N_s = 10$ the anomalous piece is more than double the value at $6/g^2 = 6.0$ whereas the value at $N_s = 18$ is roughly the same. When the anomalous term is sufficiently small compared to the bare parameter m, the chiral symmetry is effectively
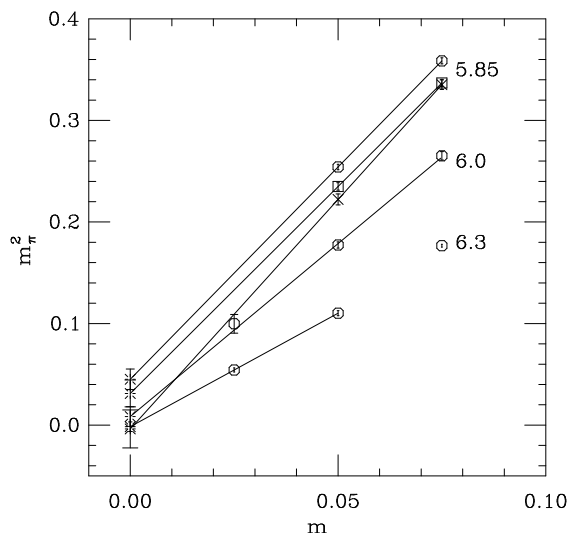
Fig. 3. The pion mass squared. $N_s = 10$ (octagons), 14 (squares), 18 (crosses).

restored. From the above, it seems we must have $\langle J_{5q}|\pi\rangle / \langle J_5|\pi\rangle \lesssim 0.1\,m$. Of course, as $m \to 0$, one must also take N_s larger, which is analogous to the situation with the ordinary spatial volume. Fewer sites in the extra dimension may be sufficient at 5.85 if M is increased still further.

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